Optimization of Steady-State Thermal Design of Space Radiators

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Equations are derived for use in optimization of fin width and tube diameter to minimize system weight of single- and double-surface space radiators. Pumping power weight penalty is included in the optimization. Incident radiation from external sources is accounted for, and calculation of required armor thickness for protection against meteoroids is included. Adaptation of the equations to numerical solution by digital computer is discussed. An illustrative problem solution is discussed in which a single-surface radiator is optimally designed for typical baseline constraints. Parametric curves are presented which illustrate the effects of off-design incident radiation levels and spacecraft internal heat loads on required fluid flow rate through the optimally designed radiator. The analysis demonstrates that required fin width and tube length are very sensitive to the level of incident radiation.

Nomenclature

A = sensitive area for meteoroid protection, ft^2
B = fin width, ft
$\bar{c}_p$ = average specific heat of radiator fluid, Btu/lb-°R
D = outside diameter of armor around tubes, ft
D_i = inside diameter of radiator tube, ft
F_f, F_t = average configuration factors, fin to space and tube to space, respectively
h = forced convection heat-transfer coefficient, Btu/hr-ft^2-°R
K' = constant: 1.09 for Al projectiles impinging on Al targets; 0.606 for iron particles impinging on iron targets
k_f, k_t = thermal conductivities of fin and tube materials, respectively, Btu/hr-ft-°R
L = radiator tube length, ft
N = number of tubes
$p(n)$ = probability that n punctures will occur in time τ over sensitive area A
Q_s = average absorbed irradiation flux from all external sources (for double-surface radiators Q_s is the sum of the average values on both sides), Btu/hr-ft^2
R_d, R_s = equivalent thermal resistances between working fluid and base of fin for double- and single-surface radiators, respectively, hr-ft^2-°F/Btu
T = radiator surface temperature, °R; T_{1m}, T_{2m} = mean values at $x = 0$, $x = (B + D)/2$, respectively (and $\Delta T_m = T_{1m} - T_{2m}$); T_{1y}, T_{2y} = local values at $x = 0$, $x = (B + D)/2$, respectively
T_b = bulk fluid temperature, °R; T_{bi} and T_{bo} = inlet and outlet values, respectively
t, t_a, t_w = fin, armor, and tube wall thicknesses, respectively, ft
v = meteoroid velocity, miles/sec, Eq. (31)
W_{eq} = radiator equivalent weight = $W_r + W_f + W_m + W_{pp}$, where W_r = basic radiator weight, W_f = fluid weight, W_m = meteoroid protection penalty, and W_{pp} = pumping power penalty, lb
$\dot{w}_T$ = radiator fluid flow rate per tube, lb/hr-tube
x, y = rectangular coordinates of radiator surface, ft
ϵ = emissivity of radiator coating
η_f, η_f' = fin effectiveness over elemental strip dy of single- and double-surface radiators, respectively: $\bar{\eta}_f, \bar{\eta}_f'$ = mean values
ξ = fin temperature gradient in x direction, Eq. (10), deg/ft
σ = Stephan-Boltzmann constant, 0.1714(10^{-8}), Btu/hr-ft^2-°R^4
τ = duration of exposure to meteoroids, days

Received April 7, 1969; revision received July 28, 1969. This work was supported by NASA under Contract NAS8-20073.

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Introduction

CURRENT emphasis on manned workshop vehicles for orbital and interplanetary space exploration has resulted in extensive efforts toward advancing the design of active thermal control systems. Radiator design has a direct influence on spacecraft configuration and structural design because of the large surface area required for heat rejection. Many of the published analytical treatments[1-5] of the heat transfer from fin-tube radiators have been concerned with the effects of certain assumptions on the analytical accuracy of the heat-transfer solution, with little attention devoted to optimization with respect to system weight (including fin width and the dependence of pumping power weight penalty on tube diameter). Reference 6 discusses structural optimization of space radiators with respect to tube spacing and fin thickness. This paper presents the derivation of equations for use in optimization of the steady-state thermal design of single- and double-surface radiators, subject to irradiation from external sources. The procedures presented are also useful for obtaining preliminary design estimates for transient heat loads through consideration of average or worst-case heat rejection requirements. A FORTRAN digital computer code incorporating the design procedures derived herein is described by Ref. 7.

The single-surface radiator (Fig. 1) would be fabricated as an integral part of the spacecraft skin structure and would radiate from the outer surface only. Double-surface radiators (Fig. 2) would be attached externally to the spacecraft

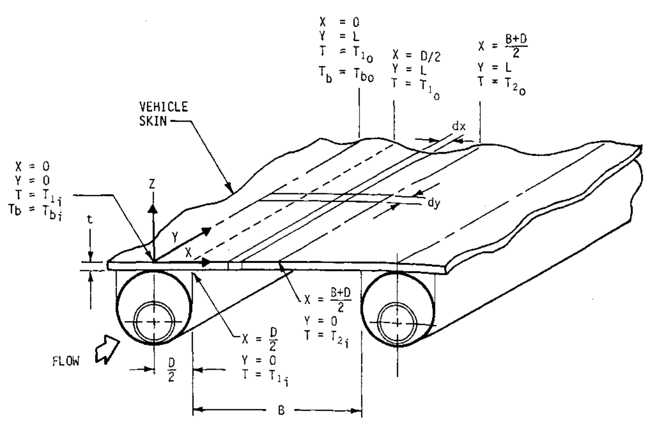

Fig. 1 Thermal model for single-surface radiator.

in the plane of the vehicle axis, and would radiate from both sides. To reduce the required surface area, the spacecraft should be oriented such that the radiator is in the plane of the solar vector during periods of heat rejection. However, the thermal analysis described herein extends the work presented in Ref. 4 to include the effect of incident solar, albedo, and planetary radiation on radiator design, because it is virtually impossible to orient the spacecraft such that incident radiation is always eliminated, especially during planetary orbit.

Thermal Analysis

Single-Surface Radiators

The tubes of the single-surface radiator (Fig. 1) are contained inside the armor thickness and vehicle skin. The derivation of the equations is based on the following assumptions: 1) heat transfer on the vehicle side is negligible; 2) $\partial T/\partial x = 0$ from $x = 0$ to $x = D/2$ or $T = T(y) = T_{1y}$ for $0 \leq x \leq (D/2)$ and $0 \leq y \leq L$; 3) $T = T(x, y)$, $(D/2) \leq x \leq (B + D)/2, 0 \leq y \leq L$; 4) inlet and outlet fluid bulk temperatures T_{bi} and T_{bo}, flow rate per tube $\dot{w}_T$, and average absorbed heat flux Q_s are known. Based on these assumptions, it is desired to calculate the optimum fin width B, tube inside diameter D_i, required tube length L, and radiator equivalent weight W_{eq}. In the process of calculating W_{eq}, the weight penalties W_m due to meteoroid armor thickness and W_{pp} due to fluid pressure drop and consequent pump weight and power supply weight must also be determined.

A steady-state heat balance on the element dy of Fig. 1 yields

$$\frac{\dot{w}_T}{2}\bar{c}_p dT_b - Q_s\left(\frac{D+B}{2}\right)dy = -\sigma\epsilon\left(T_{1y}^4\frac{D}{2} + \eta_f T_{1y}^4\frac{B}{2}\right)dy \tag{1}$$

where the fin effectiveness η_f over an incremental strip dy is expressed analytically as

$$\eta_f = \left(\int_{D/2}^{(B+D)/2} F_f T^4 dx\right)\Big/ T_{1y}^4\frac{B}{2} \tag{2}$$

The absorbed irradiation flux Q_s includes all external radiation sources, i.e., solar, albedo, planetary emission, and radiation from other structural members. For single-surface radiators the configuration factor F_f from fin to space is unity, but for double-surface radiators it must be evaluated.

Reference 4 demonstrates that an appropriately determined value of η_f is a constant mean value $\bar{\eta}_f$ over the interval of integration in the y direction. Evaluation of $\bar{\eta}_f$ is discussed later. Substituting Eq. (2) into Eq. (1) and rearranging yields

$$\frac{dT_b}{T_{1y}^4(D + \bar{\eta}_f B) - Q_s(D+B)/\sigma\epsilon} = \frac{-\sigma\epsilon dy}{\dot{w}_T\bar{c}_p} \tag{3}$$

If the radiator is constructed of a thin material of high thermal conductivity and if the fluid heat-transfer coefficient is large, it may be assumed that

$$dT_b/dy \approx dT_{1y}/dy \tag{4}$$

Substituting Eq. (4) into Eq. (3), and letting

$$a = \frac{\sigma\epsilon}{\dot{w}_T\bar{c}_p},\ b^4 = D + \bar{\eta}_f B,\ c^4 = \frac{(D+B)Q_s}{\sigma\epsilon} \tag{5}$$

Eq. (3) becomes

$$dT_{1y}/(b^4 T_{1y}^4 - c^4) = -a\,dy \tag{6}$$

Employing the method of partial fractions and integrating Eq. (16) in the y direction from $y = 0$, $T_{1y} = T_{1i}$ to $y = L$, $T_{1y} = T_{1o}$, yields the following equation for required tube length L:

$$L = \frac{1}{2abc^3}\left\{\frac{1}{2}\ln\left[\frac{T_{1i} - (c/b)}{T_{1i} + (c/b)}\right] - \frac{1}{2}\ln\left[\frac{T_{1o} - (c/b)}{T_{1o} + (c/b)}\right] + \tan^{-1}\left(\frac{bT_{1o}}{c}\right) - \tan^{-1}\left(\frac{bT_{1i}}{c}\right)\right\} \tag{7a}$$

In the limiting case of zero incident heat flux, Eq. (7a) yields the indeterminate form (0/0). Application of the L'Hospital rule yields, after three successive differentiations,

$$\lim_{Q_s\to 0} L = \frac{\dot{w}_T\bar{c}_p}{3\sigma\epsilon(D + \bar{\eta}_f B)}\left(\frac{1}{T_{1o}^3} - \frac{1}{T_{1i}^3}\right) \tag{7b}$$

which is equivalent to the relation given in Ref. 4 where incident heat flux is neglected.

Before L can be calculated it is necessary to evaluate the mean fin effectiveness $\bar{\eta}_f$, which is defined as the ratio of the heat-transfer rate from the fin to the heat-transfer rate if the fin were at the base (tube) temperature, averaged over L. Considering a steady-state heat balance on the volumetric element $tdxdy$ in Fig. 1, and assuming that conduction in the x direction only is important, the following equation results:

$$Q_s dxdy - k_f t(dT/dx)dy = -k_f t(d/dx) \times [T + (dT/dx)dx]dy + \sigma\epsilon T^4 dxdy \tag{8}$$

After simplification, Eq. (8) may be written as

$$d^2T/dx^2 = (\sigma\epsilon T^4/k_f t) - Q_s/k_f t \tag{9}$$

Integration with respect to x and application of the boundary conditions, $[dT/dx = 0,\ T = T_{2y}$ at $x = (B + D)/2]$, yield the following relation for the fin temperature gradient in the x direction:

$$dT/dx \equiv \xi = [(0.4\sigma\epsilon/k_f t)(T^5 - T_{2y}^5) - (2Q_s/k_f t)(T - T_{2y})]^{1/2} \tag{10}$$

Substituting Eq. (10) for dx into Eq. (2) and making the appropriate change of integration limits yields

$$\eta_f B = \frac{2}{T_{1y}^4}\int_{T_{2y}}^{T_{1y}}\frac{T^4 dT}{\xi} \tag{11}$$

To derive an equation for the fin width B in terms of temperature difference $(T_{1y} - T_{2y})$, Eq. (10) is integrated between the limits $[x = D/2,\ T = T_{1y}$ to $x = (B + D)/2,\ T = T_{2y}]$, which yields

$$\frac{B}{2} = \int_{T_{2y}}^{T_{1y}}\frac{dT}{\xi} \tag{12}$$

Considering steady-state heat flow from the working fluid to the outer fin surface, the following expression results from which the fin temperature T_{1y} at $x = 0$ may be calculated:

$$T_b = T_{1y} + R_s(\sigma\epsilon T_{1y}^4 - Q_s) \tag{13}$$

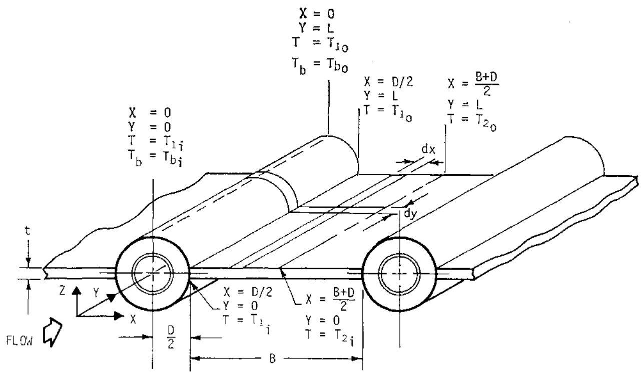

Fig. 2 Thermal model for double-surface radiator.

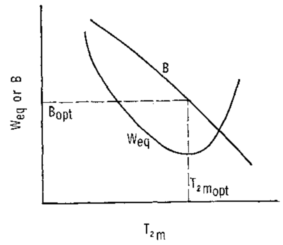

Fig. 3 Example of radiator fin width optimization.

where R_s is the equivalent thermal resistance (including boundary layer, metallic conduction and interface contact resistances) between the fluid and the outer fin surface for single surface radiators.

Reference 4 recommends that the limits of integration T_{1_y} and T_{2_y}, of Eqs. (11) and (12) be replaced by T_{1_m} and T_{2_m}, respectively, where

$$T_{1_m} = [(T_{1_i}^4 + T_{1_o}^4)/2]^{1/4} \tag{14}$$

T_{2m} is defined similarly in terms of T_{2_i}, T_{2_o}. T_{1_i}, and T_{1_o} may be calculated by iteration from Eq. (13). Thus, the local fin effectiveness η_f is replaced by the mean value $\bar{\eta}_f$ which is evaluated at the fourth root of the mean fourth power of T_{1_y}.

Integration of Eqs. (11) and (12) may be accomplished numerically using Simpson's rule except near the point ($T = T_{2m}$), where a singularity exists. Near ($T = T_{2m}$) the integral is evaluated by substituting an approximate function which may be integrated directly.

Through algebraic manipulations, the integrand of Eq. (11) may be expressed as

$$I_1 = \left[\frac{2}{k_f t}(T - T_{2m})\right]^{-1/2} \left\{T^4 \Big/ \left[\frac{\sigma\epsilon}{5}(T^4 + T^3 T_{2_m} + T^2 T_{2_m}^2 + T T_{2_m}^3 + T_{2_m}^4) - Q_s\right]^{1/2}\right\} \tag{15}$$

where T_{2_y} is replaced by T_{2_m} as discussed previously. As $T \rightarrow T_{2m}$, the term enclosed in braces approaches a constant value. Therefore, over a small interval near ($T = T_{2m}$), the integrand may be approximated by

$$I_1 \simeq K/(T - T_{2m})^{1/2} \tag{16}$$

where

$$K = T_{2_m}^4/[(2/k_f t)(\sigma\epsilon T_{2_m}^4 - Q_s)]^{1/2}$$

This approximate function I_1 may be integrated directly. Hence, the integral of Eq. (11) is evaluated in two additive parts. Between the limits T_{2m} and T^*, where T^* is very near to T_{2m}, the approximate function I_1 is integrated directly. Between the limits T^* and T_{1_m} the original function is integrated numerically using Simpson's rule. Equation (11) is thus written in the approximate form used for computation as

$$\bar{\eta}_f B \simeq \frac{4K(T^* - T_{2m})^{1/2}}{T_{1_m}^4} + \frac{2}{T_{1_m}^4}\int_{T^*}^{T_{1_m}} \frac{T^4 dT}{\xi_m} \tag{17}$$

where ξ_m is ξ [Eq. (10)] with T_{2_y} replaced by T_{2_m}, and T^* is chosen near enough to T_{2m} to produce negligible error

$$\frac{B}{2} \approx \frac{2K(T^* - T_{2m})^{1/2}}{T_{2_m}^4} + \int_{T^*}^{T_{1_m}} \frac{dT}{\xi_m} \tag{18}$$

Equations (7, 13, 17, and 18) can now be solved simultaneously to determine radiator thermal performance. The recommended procedure to optimize fin width and tube diameter is as follows.

1) Determine T_{1_i} and T_{1_o} from Eq. (13) using the inlet and outlet bulk fluid temperatures.

2) Calculate T_{1_m} from Eq. (14); assign several specific values to T_{2_m} and perform the following steps (3–5) for each value. (The selected values of T_{2_m} must be chosen such that the radical terms in the denominators of Eqs. (17) and (18) are positive.)

3) Calculate B from Eq. (18).

4) Calculate $\bar{\eta}_f$ by integrating Eq. (17) between limits T_{1_m} and T_{2_m}, and then dividing by B.

5) Calculate L from Eq. (7a) or (7b).

6) Calculate radiator equivalent weight W_{eq} including hardware, pumping power penalty and heat transport fluid, for each T_{2_m}, thus establishing relationships as illustrated by Fig. 3. The fin width B corresponding to the minimum equivalent weight is "optimum."

7) Determine the optimum radiator dimensions for a given tube diameter. To determine the optimum tube diameter, steps (1–6) are repeated for several specified standard diameters, and the diameter yielding the minimum W_{eq} is selected.

If B is specified as a design stipulation, the procedure is modified as follows: 1) calculate T_{1_i} and T_{1_o} from Eq. (13), and T_{1_m} from Eq. (14); 2) determine T_{2_m} by iteration using Eq. (18); 3) calculate $\bar{\eta}_f$ B from Eq. (17); 4) calculate the tube length L from Eq. (7a) or (7b) and the weight; 5) repeat the previously specified step (7).

Double-Surface Radiators

The tubes of the double-surface radiator (Fig. 2) are joined and supported by the fins, and heat is radiated from both sides. The assumptions given for the single-surface radiator analysis apply, in addition to the following: 1) For each value of B/D the radiation configuration factor from the tube to space F_t is taken as the average value around a quadrant of the tube diameter extending from the base of the fin to the point $\pi/2$ rad away; 2) the configuration factor F_f from the fin to space is taken as the weighted average between the base of the fin and center of the fin for each B/D; 3) configuration factors are assumed to be identical on both sides of the fin, and longitudinal variations in F_t and F_f are neglected; 4) for double-surface radiators, the average absorbed irradiation Q_s is the sum of the average values on both sides; 5) radiation heat exchange between the tube and fin is negligible.

Writing a heat balance on the element dy of Fig. 2,

$$\frac{\dot{w}_T}{2}\bar{c}_p dT_b - Q_s\left(\frac{B}{2} + \frac{\pi D}{4} - \frac{t}{2}\right)dy = -\sigma\epsilon\left[T_{1_y}^4\left(\frac{\pi D}{2} - t\right)F_t + \bar{\eta}_f' T_{1_y}^4 B\right]dy \tag{19}$$

Letting

$$a' = \frac{2\sigma\epsilon}{\dot{w}_T \bar{c}_p'},\quad b'^4 = \left(\frac{\pi D}{2} - t\right)F_t + \bar{\eta}_f' B$$
$$c'^4 = \left(\frac{B}{2} + \frac{\pi D}{4} - \frac{t}{2}\right)\frac{Q_s}{\sigma\epsilon} \tag{20}$$

Table 1 Baseline conditions for example single-surface radiator

Spacecraft heat load	50,000 Btu/hr
Average absorbed irradiation flux	80 Btu/hr-ft²
Heat transport fluid	FC-75
Inlet fluid temperature	100°F
Specified tube length	68 ft
Surface emissivity	0.9
Fin thickness	$\frac{1}{16}$ in.
Fin and meteoroid armor material	Aluminum
Tube material	Stainless steel
Power-supply weight penalty	857 lb/kw
Tube wall thickness	0.015 in.

the aforementioned integration procedure for the single-surface radiator may be executed to obtain L,

$$L = \frac{1}{2a'b'c'^3}\left[\frac{1}{2}\ln\left(\frac{T_{1_i} - c'/b'}{T_{1_i} + c'/b'}\right) - \frac{1}{2}\ln\left(\frac{T_{1_o} - c'/b'}{T_{1_o} + c'/b'}\right) + \tan^{-1}\frac{b'T_{1_o}}{c'} - \tan^{-1}\frac{b'T_{1_i}}{c'}\right] \quad (21a)$$

In the limiting case of zero incident heat flux ($Q_s = 0$), application of L'Hospital's rule yields

$$\lim_{Q_s \to 0} L = \frac{\dot{w}_T \bar{c}_p}{6\sigma\epsilon\,[(\pi D/2 - t)\,F_t + \bar{\eta}_f' B]}\left(\frac{1}{T_{1_o}{}^3} - \frac{1}{T_{1_i}{}^3}\right) \quad (21b)$$

To evaluate $\bar{\eta}_f'$, a steady-state heat balance is written as before for element $dx\,dy$ of Fig. 2,

$$Q_s dx dy - k_f t\frac{dT}{dx}dy = -k_f t\frac{d}{dx}\left(T + \frac{dT}{dx}dx\right)dy + 2F_f\sigma\epsilon T^4 dx dy \quad (22)$$

Simplifying and rearranging,

$$\frac{d^2T}{dx^2} = \frac{2F_f\sigma\epsilon T^4}{k_f t} - \frac{Q_s}{k_f t} \quad (23)$$

Equation (23) is now integrated and the boundary condition described for Eq. (9) is applied to obtain the following expression for fin temperature gradient in the x direction:

$$dT/dx \equiv \zeta = [(0.8\sigma\epsilon F_f/k_f t) \times (T^5 - T_{2_y}{}^5) - 2Q_s/k_f t(T - T_{2_y})]^{1/2} \quad (24)$$

Substituting Eq. (24) for dx into Eq. (2), with the appropriate change of limits, and substituting T_{1_m} for T_{1_y} and T_{2_m} for T_{2_y} in ζ, as discussed previously, yields

$$\bar{\eta}_f' B = \frac{2F_f}{T_{1_m}{}^4}\int_{T_{2_m}}^{T_{1_m}} \frac{T^4 dT}{\zeta_m} \quad (25)$$

Integrating Eq. (24), we find

$$\frac{B}{2} = \int_{T_{2_m}}^{T_{1_m}} \frac{dT}{\zeta_m} \quad (26)$$

Equations (25) and (26) must be integrated numerically, as described previously for the single-surface radiator; because no feasible approach could be found for exact integration. Near ($T = T_{2_m}$), the integrand of Eq. (25) may be approximated as

$$I_2 \simeq J/(T - T_{2_m})^{1/2} \quad (27)$$

where

$$J = T_{2_m}{}^4/[(2/k_f t)(2\sigma\epsilon F_f T_{2_m}{}^4 - Q_s)]^{1/2}$$

The function I_2 may be integrated directly.

As described earlier for the single-surface radiator, the integral of Eq. (25) is evaluated in two additive parts, and Eq. (25) is written in computational form as

$$\bar{\eta}_f' B = \frac{4F_f J(T^* - T_{2_m})^{1/2}}{T_{1_m}{}^4} + \frac{2F_f}{T_{1_m}{}^4}\int_{T^*}^{T_{1_m}} \frac{T^4 dT}{\zeta_m} \quad (28)$$

where T^* is chosen near enough to T_{2_m} to produce negligible error.

In a similar manner, Eq. (26) may be approximated as

$$\frac{B}{2} = \frac{2J(T^* - T_{2_m})^{1/2}}{T_{2_m}{}^4} + \int_{T^*}^{T_{1_m}} \frac{dT}{\zeta_m} \quad (29)$$

From a steady-state heat balance on an individual tube, assuming no temperature gradient around the tube, the following expression results for the fin base temperature T_{1_y} in terms of the bulk fluid temperature for double-surface radiators:

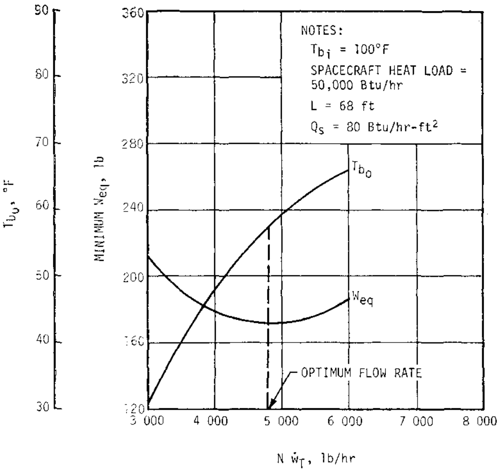

Fig. 4 Optimization of fluid flow rate.

$$T_b = T_{1_y} + R_d(F_t\sigma\epsilon T_{1_y}{}^4 - Q_s/2) \quad (30)$$

where R_d is the equivalent thermal resistance between the fluid and the base of the fin for double-surface radiators. To determine the average view factor of the tube F_t the data of Ref. 4 may be used.

Solution of these equations proceeds as described for corresponding equations of the single-surface radiator.

Meteoroid Protection and Pumping Power Penalties

The procedure recommended by Bjork[8] for calculating the required armor thickness t_a around the tubes is used. His general formula for calculating t_a for a sensitive area A exposed for a time period τ is

$$t_a = 2.15(10^{-4})K'v^{1/3}\left\{\frac{A\tau}{\ln[1/p(0)]}\right\}^{0.3} \quad (31)$$

where the recommended average meteoroid velocity v for calculations is 18.64 miles/sec. Solution of Eq. (31) must be accomplished by iteration, since A depends on t_a. (Other data and theories[9,10] differ somewhat from this criterion; the meteoroid protection criterion should be reevaluated and modified as improved techniques become available.)

The electrical power required to pump the working fluid may be calculated from well-known pump design equations as a function of fluid flow rate, density, pressure drop, and pump efficiency. To estimate pump weight, Ref. 11 recommends a specific weight factor of 18 lb/kw of delivered power. Since values of pumping power are typically of the order of 0.1 kw, the weight of the circulating pump is very small. However, a large weight penalty must be paid in terms of the spacecraft power supply to furnish even a small amount of pumping power. Typical specific weights for spacecraft solar cell/battery power supply systems are 700–900 lb/kw.[12]

Sample Problem

To demonstrate the utility of this analysis a single-surface radiator was designed for a typical set of baseline conditions (Table 1) using a computer program[7] that contains ten options to provide analytical flexibility.

In establishing the optimum radiator design for the conditions in Table 1, computer runs with various fluid flow rates were processed to determine the optimum $N\dot{w}_T$. For each $N\dot{w}_T$, B and D_i were optimized to yield minimum W_{eq}.

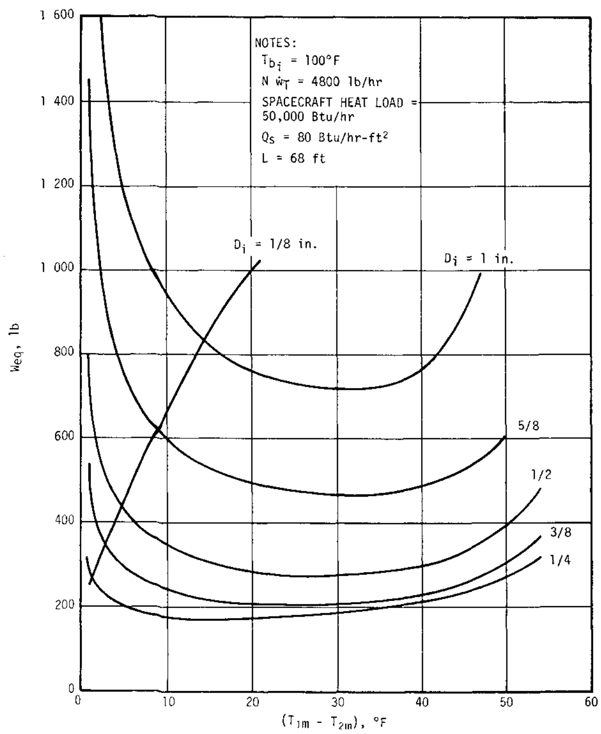

Fig. 5 Optimization of tube diameter and mean temperature difference between tube and center of fin.

The variations in minimum W_{eq} and T_{b_o} with $N\dot{w}_T$ are indicated in Fig. 4. The optimum flow rate for the specified heating conditions is 4800 lb/hr. In some cases selection of $N\dot{w}_T$ is dictated by a desired T_{b_o}, e.g., to supply a cabin heat exchanger. Since no restrictions were placed on outlet temperature in the example problem, the optimum flow rate of 4800 lb/hr was selected for further analysis. As shown in Fig. 4, this $N\dot{w}_T$ yields $T_{b_o} \simeq 57°F$.

The procedures followed by the computer program in optimizing B and D_i at the optimum flow rate are illustrated by Figs. 5 and 6. Figure 5 shows W_{eq} vs $(T_{1_m} - T_{2_m})$ for the six selected tube diameters. The optimum D_i is 0.25 in., and the optimum mean tube-center fin temperature difference lies between 10° and 20°F. The computer program's discrete-point optimization procedure selected 20°F as the optimum value, although the design using a 10°F difference is less than 1 lb heavier than the chosen design. Figure 5 reveals that, because of excess pumping power penalty W_{pp}, a D_i of $\frac{1}{8}$ in., would only be practical at this flow rate for very small temperature differences and hence a large number of tubes N. As $(T_{1_m} - T_{2_m})$ and B are increased, with fixed flow rate $(N\dot{w}_T)$ and tube length, N decreases, causing an increase in $\dot{w}_T$, pressure drop and hence W_{pp}.

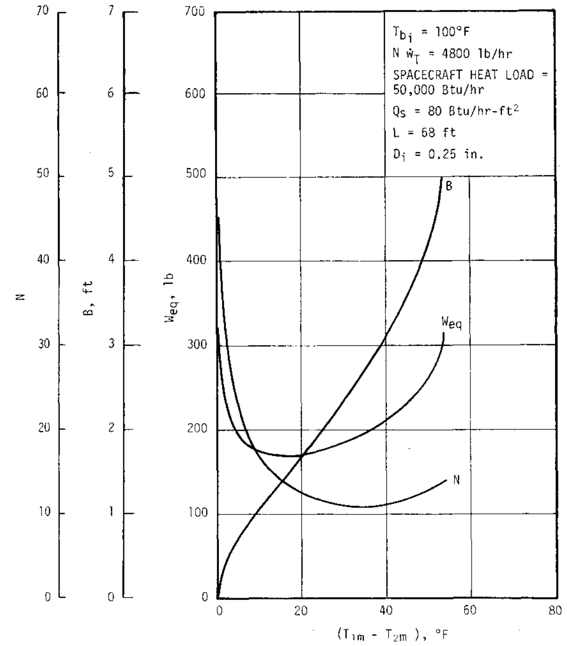

Fig. 6 Design parameters as a function of mean tube-center fin temperature difference for optimum tube diameter.

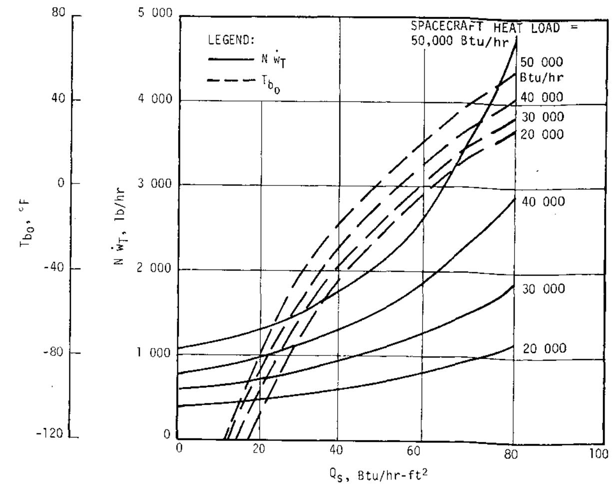

Fig. 7 Required fluid flow rate and resulting outlet temperature for off-design values of absorbed irradiation and cabin heat load.

The variations of B, N, and W_{eq} with $(T_{1_m} - T_{2_m})$ are shown in Fig. 6 for the optimum D_i of 0.25 in. and optimum flow rate of 4800 lb/hr. As ΔT_m and B increase, W_{eq} passes through a minimum at a mean ΔT of approximately 20°F and a fin width of 1.68 ft. As discussed earlier, a fin width as small as 1 ft could be employed with little weight penalty. It is noted that as ΔT_m and fin width are increased further, the required number of tubes decreases, passes through a minimum, and then increases again. This increase is caused by increased total surface area requirements due to the downward trend in fin efficiency with increased fin width.

The selected optimum radiator design is described in detail by Table 2. As indicated no armor thickness is required for meteoroid protection. This is because the tubes are attached underneath the vehicle skin, and, in this case, the computer

Table 2 Description of optimum radiator design for baseline conditions

Design parameter	Value
Fluid flow rate (FC-75)	4800 lb/hr
Outlet temperature	57°F
Number of tubes	13
Optimum tube diameter	0.25 in.
Optimum fin width	1.68 ft
Total radiator width	22.2 ft
Total surface area	1510 ft²
Required armor thickness around tube	Zero
Fluid pressure drop	9.8 psi
Required pumping power	39 w
Weight summary	
Item	Weight, lb
Radiator weight (dry)[a]	104.8
Fluid weight	31.2
Power supply weight penalty	331.
Pump weight	2.4
Total equivalent weight	171.5

[a] Includes estimate of manifold weight; excludes radiator fin weight.

program determined that the input fin thickness of 0.0625 in. provided sufficient meteoroid protection for a 1-yr mission. It is noted that the radiator weight excludes the fin weight, since the vehicle skin serves as the fin.

After the optimum radiator design was established for the baseline conditions, additional computer runs were processed to determine the required variation in fluid flow rate to accommodate off-design variations in absorbed irradiation and spacecraft internal heat load. The necessary fluid flow rate and corresponding tube-center fin temperature difference to provide the input fin width and tube length were determined by Newton-Raphson iteration within the program. The variation in fluid flow rate and outlet temperature vs absorbed irradiation is shown in Fig. 7 for various values of spacecraft internal heat load. Figure 7 illustrates the strong importance of the level of absorbed irradiation on radiator design. The required flow rate decreases rapidly as absorbed irradiation decreases below 80 Btu/hr-ft^2. Likewise, for a fixed flow rate, the required fin width and tube length are very sensitive to variations in absorbed irradiation. Figure 7 shows that, in general, very low outlet temperatures result from decreasing the flow rate to accommodate reduced values of absorbed irradiation with fixed radiator dimensions and a given internal heat load. This fact must be accounted for in over-all thermal control system design by mixing the low-temperature fluid with higher-temperature bypass or regenerative flow to produce the desired temperature at the inlet to the cabin heat exchanger or cold plates.

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